Let $p(x)$ be a polynomial and let c be a number.

Remainder Theorem:
If you divide $p(x)$ by $x - c$, then the remainder will be $p(c)$.

Factor Theorem:
If $p(c) = 0$, then $x - c$ is a factor of $p(x)$.
Conversely, if $x - c$ is a factor of $p(x)$, then $p(c) = 0$.

Definition: c is a *root* of $p(x)$ if and only if $p(c) = 0$.

1. Divide using polynomial long division. Identify the divisor, quotient, and remainder.

   (a) $\dfrac{2x^3 + 4x^2 - 5}{x + 3}$ (b) $\dfrac{2x^3 - 4x + 7x^2 + 7}{x^2 + 2x - 1}$ (c) $\dfrac{4x^3 - 2x^2 - 3}{2x^2 - 1}$

2. Divide using synthetic division. Identify the divisor, quotient, and remainder.

   (a) $\dfrac{x^3 - 5x^2 + 3x + 7}{x - 3}$ (b) $\dfrac{3x^3 + 5x - 1}{x + 1}$ (c) $\dfrac{4x^3 - 8x^2 - x + 5}{2x - 1}$

3. What is the remainder if you divide each of the following $p(x)$ by $x - 2$? Is $x - 2$ a factor of $p(x)$?
   Hint: Use the Remainder and Factor Theorems.

   (a) $p(x) = -2x^3 + 5x - 1$
   (b) $p(x) = 3x^2 - x^5 + 7x + 6$
   (c) $p(x) = x^4 + 6x^3 - x^2 + 10$

4. Find all roots of $p(x)$.

   (a) $p(x) = x^3 - 19x - 30$, given that 5 is a root
   (b) $p(x) = x^3 - 6x + 4x^2 - 24$, given that -4 is a root
   (c) $p(x) = 8x^3 - 10x^2 - x + 3$, given that $\frac{3}{4}$ is a root
   (d) $p(x) = x^3 - 6x^2 + 11x - 6$
   (e) $p(x) = x^4 + x^3 - 7x^2 - x + 6$

Hint: Use the theorems and definition to help you find at least one root for (d),(e).