Set Theory: Venn Diagrams & Subsets

1. Consider the set $A = \{1, 3, 5, 7, 9\}$.

   (a) Determine the number of proper subsets of A.
   $2^5 - 1 = 31$

   (b) What's the only non-proper subset of A?
   A

   (c) Consider the set $B = \{0, 1, 3\}$. Is B a subset of A? Explain why or why not.
   No, $0 \in B, 0 \notin A$

2. Suppose you have a collection of dollar bills, one twenty, one ten, one five, and one single. How many different sums of money can you make using just these bills? (Note: You don't have to use all of them).
   $2^4 = 16$

3. There are nine students in a math club. At least one of these students must show up to advertise the club at freshman orientation. How many different subsets of the math club could possibly show up?
   2^9

4. Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Find the complement of each of the following sets.

   (a) U
   $U' = \emptyset$

   (b) $\{1, 2, 3, 4, 5\}$
   $\{0, 6, 7, 8, 9\}$

   (c) $\emptyset$
   U

   (d) $\{0, 2, 4, 6, 8\}$
   $\{1, 3, 5, 7, 9\}$

5. In discovering the expression 2^n for finding the number of subsets of a set of n elements, it's easy to see that for the first few values of n, increasing the number of elements by 1 doubles the number of subsets. We will prove this formula in general for any value of n. Let A be a set with n elements and s subsets. Let us add an additional element, let's call it e, to the set A. Note we now have a new set, we can call this B, with $n+1$ elements.
While general solutions are given, it is highly advised that you explore and understand the solutions

   (a) How many subsets of B do not contain e?
   2^n (all subsets of A, which contained n elements.

   (b) How many subsets of B do contain e? (Hint: these are all the subsets that don't contain e with an additional element e).
   2^n, each of the subsets above, except now each contains element e

   (c) What is the total number of subsets of B?
   $2^n + 2^n = 2(2^n) = 2^{n+1}$

   (d) What do you conclude?